

A program for a problem free Cosmology within a framework of a rich class of scalar tensor theories

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Abstract

A search for a problem free cosmology within the framework of an effective non - minimally coupled scalar tensor theory is suggested. With appropriate choice of couplings in variants of a Lee - Wick model [as also in a model supporting Q - ball solutions], non topological solutions [NTS's], varying in size upto 10 kpc to 1 Mpc can exist. We explore the properties of a “toy” Milne model containing a distribution of NTS domains. The interior of these domains would be regions where effective gravitational effects would be indistinguishable from those expected in standard Einstein theory. For a large class of non - minimal coupling terms and the scalar effective potential, the effective cosmological constant identically vanishes. The model passes classical cosmological tests and we describe reasons to expect it to fare well as regards nucleosynthesis and structure formation.

I. Introduction

A description of a Friedman - Robertson - Walker [FRW] universe within the framework of general theory of relativity [GTR] has its own peculiar problems. Besides having an initial singularity, a lack of a consistent quantum mechanical framework and a lack of a consistent account of large scale structure formation in the theory, the observed large scale homogeneity and isotropy can not be dynamically generated in standard GTR due to the so called “horizon problem”. The stability of the FRW solution, moreover, requires a fine tuning of the density parameter in the theory unless it is exactly equal to the critical density. This is referred to as the flatness problem. A resolution of these problems using inflation has developed into a state of art over the last almost two decades. During the inflationary epoch, the energy density of the universe is dominated by the vacuum energy of an inflaton field while the scale factor expands superluminally. With a sufficiently large interval of exponential inflation, a small causally connected region of the universe grows sufficiently large to account for the the observed homogeneity and isotropy of the universe. In most versions, however, this paradigm requires a very special profile for the effective potential for scalar field $[s]$ in the model on account of constraints on inflation [1]. The requirement of sufficient inflation and CMB anisotropy limits the density fluctuations [2] and in turn constrains the inflaton field potential to be very flat. For a general class of inflationary models involving a slow rolling field [including new [3], chaotic [4] and multiple field [5] inflation], one needs the inflaton potential to satisfy: $\Delta V/(\Delta\phi)^4 \leq 10^{-6} - 10^{-8}$ [6]. Where ΔV and $\Delta\phi$ are changes in the potential and the field during the slow rolling. For a quartic coupling $[V \approx \lambda\phi^4]$ for example, this presents a constraint $\lambda \leq 10^{-6} - 10^{-8}$ on the coupling constant. Small couplings at tree level are unnatural in general as they require fine tuning to cancell large radiative corrections. Standard convexity and triviality theorems in quantum field theory also cast a doubt on whether such an effective potential can at all be realised. Most versions of an appropriate inflation are further heavily constrained by baryon asymmetry constraints [the graceful exit and the reheating problems (see eg.[7] for a recent review)].

Assuming that these formal problems would be resolved, inflationary scenarios ensure the favoured value for the density parameter of the FRW universe viz.: the closure density, in turn ensuring the density parameter $\Omega = 1$. This defines what is now regarded as the so called Standard - Big - Bang model [SBB]. Over the last decade, the SBB has suffered a lot of stress even as far as the post - inflationary empirical observations are concerned. There has been a steady growth of evidence indicating that Ω over scales of a giga parsec and more is significantly less than one. Figures of .2 to .8 have been quoted in literature. The flatness problem [fine tuning] then stares SBB in the face. This is not withstanding models [8] contrived to yield Ω less than unity as a result of multi - inflating epochs that judiciously combine an epoch of old inflation followed by a new - inflation wherein Ω 's approach to unity is cut short arbitrarily. These models include “extended and hyperextended” inflationary models [9], in which both the inflaton and the Brans - Dicke field come with their respective potentials and a proliferation of parameters. With better instrumentation and observations, the parameter space of the post inflationary

SBB universe is shrinking. The worst constraints come from age estimates of old clusters in comparison with age estimates of the universe from the measurement of the Hubble parameter. There is further evidence of excessive baryonic dark matter, inconsistent with SBB nucleosynthesis, from the intensity of x-ray from centres of clusters of galaxies. Recent magnitude - redshift relations based on type SN1A observations [10], rule out the $\Omega \geq 1$ models. It has been proposed that a small cosmological constant be incorporated in the model. While this would not do away with the fine tuning problem, the resolution that it may achieve is seriously in doubt. The SBB may well be on the verge of a crisis.

To summarise: though inflation can resolve the horizon, flatness and the monopole problems and has a promise to give an ansatz for primordial density fluctuations, it is still too early and naive to defend this paradigm as a faith. For one, the inflaton field requires a fine tuning of its dynamical parameters leaving hardly any “naturalness” in the model. Further, a transition from an almost empty inflated patch to a matter filled universe (the reheating and the graceful exit problems) can not be dynamically realised in most versions of the inflationary scenario. Most versions of inflation find the smallness of the cosmological constant an embarrassment. The “natural” value of the cosmological constant in most unification schemes, the inverse square of the Planck length, differs from cosmological observational bounds by some 120 orders of magnitude ! In its most acceptable form, it would be fair to view inflation as a paradigm that relates large scale properties [over distances greater than the the hubble length scale at the last scattering surface] to the physics at Planck energy scale - either through the peculiarities of a finely tuned [contrived] scalar potential or through quantum gravity. As one hardly has any testable attributes of Planck physics on laboratory scales, there is a feeling of arbitrariness and little predictive power [11] in the exercise of fitting any cosmological attribute to a corresponding attribute of the inflaton effective potential.

Features of inflation have been extended to scalar tensor theories in general. These include the Brans - Dicke scalar tensor theory together with a separate inflaton field mentioned earlier. Recent interest in these theories motivates from string theory. It is believed that so far the only consistent theory of quantum gravity is string theory [12]. The low energy effective action that follows from string theory has the form of a scalar tensor theory of gravity with non - trivial coupling of the dilaton to matter [13]. The entire excercise of constraining the extra parameters of the Brans - Dicke (dialaton) field, with or without an further parameters of an inflaton field, has been extensively described in literature [14]. Rather than looking for empirical fits in the framework of proliferating parameters, it would be a relief if basic cosmic attributes like the absence of the horizon and the fine tuning problems were to be explained for a general equation of state of matter (including any “dialaton” or an “inflaton” fields). It is with this hope that we proceed to describe features of a toy model in the following:

A dynamic realisation of large scale homogeneity and isotropy in a FRW universe for an arbitrary equation of state requires the scale factor to evolve as t^n with $n \geq 1$. It turns out that within the framework of a general scalar tensor theory and for a general equation of state of matter, the only possible value is $n = 1$ [Milne model[15]]. As we shall see in sec III, such a coasting cosmology is not excluded by classical cosmological tests [16]. Such a scaling, however, can only be possible in general in a model in which

long distance gravity vanishes. [There are special solutions in Brans - Dicke theory that can also support such a scaling [17]. However: (a) these do not stand upto cosmological constraints and (b) these are not stable to perturbations of the parameters of the theory]. The most crucial requirement for such an idea would be to specify an ansatz that could account for a spatial variation of the gravitational constant that vanishes outside compact domains and be uniform in the interior. The ansatz ought to generate effective gravitating domains upto say 1Mpc in size.

In this article we outline an effective gravity model of a scalar tensor [ST] theory of gravity in which a Higgs field ϕ is also coupled to the scalar curvature R through a function $U(\phi)$ which diverges at some point that we take as $\phi = 0$ Thus: $U(\phi \rightarrow 0) \rightarrow \infty$. With the Higgs generating masses for Fermions, non - topological soliton solutions [NTS's] arise in such theories [18]. We show that each NTS is a domain that separates the exterior region, where the non minimal coupling with the Ricci scalar is chosen to diverge, from the interior where the scalar field could be arrested to an arbitrary value - depending on the profile of the effective potential and the non - minimal coupling. The interior is thus a region with an effective gravitational constant and an effective cosmological constant determined by the interior values of the non minimal coupling term $U(\phi)$, and the scalar effective potential $V(\phi)$ respectively. The essential point is that we expect the scalar field in the interior of larger and larger stable NTS domains to approach a value that has smaller and smaller $|V(\phi)|$. In this limit, the effective gravitational constant would approach a universal value inside all large NTS's - determined by a value ϕ_{in}^o where the effective potential vanishes. This would be an effective solution to the cosmological constant problem. The effective cosmological constant would vanish outside a NTS (as the effective gravitational constant is zero there) and be near zero in the interior of all large NTS's. Similar solutions are also possible in a theory in which the scalar curvature non - minimally couples to an invariant function of a multi - component scalar field. Such possibilities have been explored recently in a search of alternative inflationary models [19]. If the invariant NMC again diverges at $|\phi| \equiv \sqrt{\phi_1^2 + \phi_2^2} = 0$ for [say] an SO(2) invariant field non - topological soliton solutions [NTS's] again exist [Q - balls] with properties similar to those described above for the analogous Lee-Wick constructs. The interior is again a region with an effective gravitational constant and an effective cosmological constant determined by the interior values of the non minimal coupling term $U(|\phi|)$, and the scalar effective potential $V(|\phi|)$ respectively. We expect each NTS domain to expand while conserving the charge and the energy of the NTS. This necessarily implies that the expansion would be accompanied by a drift of $|\phi|$ to a value $|\phi_o|$ where $V(|\phi|)$ vanishes. As $|\phi| \rightarrow |\phi_o|$, the volume of the domain becomes large. In this limit, the effective gravitational constant would approach a universal value inside all NTS's - determined by a value $|\phi_o|$ where the effective potential vanishes.

The program essentially requires non - minimal coupling [NMC]. There is no compelling principle to constrain the coupling of a function of a scalar field with the scalar curvature. Conformal coupling for a single component scalar field: $U(\phi) = \phi^2/6$ is known [20] to give decent renormalisable properties of the stress energy tensor. Zee [21] and earlier, Deser [22] have considered NMC's to generate effective Brans - Dicke like theories virtually indistinguishable from general theory of relativity [GTR] at low energies. Dolgov

[23] has explored [though not successfully] a rising NMC as a mechanism to dynamically reduce the effective cosmological constant. Others [19] have used invariant NMC's in multi-component extended inflationary models. Madsen [24] has extensively reviewed properties of a large class of NMCS. There seems to be no consensus on any particular principle that may be used to specify the NMC. For our purpose we propose an ansatz that leaves the form for the NMC unspecified as an otherwise arbitrary function $U(\phi)$ that diverges at $\phi = 0$ and has a sufficiently large gradient in an open interval containing $\phi = \phi_{in}^o$ where the effective potential vanishes. We require the model to support NTS's. This is what is outlined in the next section where we describe constraints on a scalar field theory in order that it generate an effective theory of gravitation indistinguishable from GTR. In the class of ST theories considered, we establish the existence of solutions across the boundary of the NTS - connecting to the flat spacetime in the exterior of the NTS.

Section III, translates some of the aspects of standard model in the framework of the “toy” model described here and summarises the predictions of the model. In the appendix we describe the essential properties of a scalar tensor theory. Expressions for a conserved energy - momentum pseudo - tensor are derived for a general non - minimally coupled ST theory.

II. A Milne model with a difference:

We revisit the favourite model of Milne [15], who considered the evolution of an exploding universe from a highly correlated state [a vanishingly small ball of particles of infinite density] localised near the neighbourhood of the point $x = y = z = 0$ at $t = 0$, in a Minkowski spacetime. At any later time, the universe consists of an isotropically expanding swarm of particles of all speeds bounded by the speed of light. Special relativity tells us that the universe looks the same (isotropically expanding ball bounded by a light front) in all Lorentz frames that coincide at the origin $x = y = z = 0$ at $t = 0$. In this sense the universe is homogeneous and isotropic about every such Lorentz observer - strictly obeying the so called cosmological principle. This is manifestly obvious in co - moving coordinates in which the Minkowski metric describing the expanding ball reduces to

$$ds^2 = dt^2 - t^2 \left[\frac{dr^2}{1+r^2} + r^2 d\Omega^2 \right] \quad (2.1)$$

This has the form of an open FRW metric with the scale factor $a(t) = t$. The most appealing feature of this model is that at any time $t > 0$, every co - moving observer can see the entire universe : there is no horizon in the model.

$$\int_0^t \frac{dt}{a(t)} = \infty \quad \forall t > 0 \quad (2.2)$$

There is no flatness problem either as the rate of expansion of the universe is not constrained by any “critical density” parameter. This however, may be regarded as a trivial solution to the flatness problem. Eqn[2.1] is a solution to Einstein’s equations only if the product of the gravitational constant and the density $G\rho$ vanishes. In canonical Einstein theory, the Milne metric can thus only be a solution for an empty ($\rho = 0$) universe. The model is thus put away without any further ado. However, one can try to make out a case for a search for models in which the universe coasts freely over large distances and has gravitating domains localised in pockets. In this article we report on our study of requirements on classes of ST theories in which the large scale dynamics of a non - empty universe is described by eqn[2.1] on account of the vanishing of the effective long - distance gravitational constant.

Consider for example, a ST theory characterised by a non - minimal coupling of a scalar field ϕ with the scalar curvature R , through an arbitrary function $U(\phi)$, in an effective action:

$$S = \int \sqrt{-g} d^4x [U(\phi)R + \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi) + L_m] \quad (2.3)$$

Here $V(\phi)$ is the scalar effective potential and L_m is the contribution from the rest of the [matter] fields. Throughout our analysis we treat ϕ as an effective classical field. Let $V(\phi)$ be inclusive of an additive constant whose source could be the characteristic cut - off mass scale that appears when we renormalise any quantum [matter] field. It could even be an arbitrary integration constant. It is this constant that manifests itself as an arbitrary cosmological constant in the theory. The essential features of such a theory are described in the Appendix A. As shown, the stress energy tensor for the rest of the matter fields has a vanishing co - variant divergence - in concordance with the equivalence principle. We shall also find it essential to include, in L_m , a coupling between the scalar field and a fermion field - giving rise to an effective mass for the fermion determined by the local value of the scalar field.

If the model can consistently support a divergent $U(\phi) \longrightarrow \infty$ at (say) $\phi = 0$, flat space is a solution for $\phi = 0$ for an arbitrary $V(\phi = 0)$. This gets rid of the cosmological constant problem. However, this is a trivial cure as the blowing up of $U(\phi)$ implies a vanishing of the effective gravitational constant, and we all know [25] that, but for gravitation, the additive constant in the effective potential has no dynamical role in physics. What we want is a cure to the cosmological constant problem in the presence of gravitation.

We look for non trivial solutions with $U(\phi \longrightarrow 0) \longrightarrow \infty$. In flat spacetime, [with the $U(\phi)R$ term missing in eqn(2.3)], fermion number conservation plays a key role in the existence and stability of non - trivial, NTS’s. Such solutions were suggested by Lee and Holdom [18] for a potential $V(\phi)$ with a degenerate minima and the analysis has been extended to non degenerate effective potentials [26]. The analysis easily extends to NTS’s which are 3 - dimensional bounce solutions and can be summarised by referring to a variant of the Lee - Wick model of interacting fermions ψ and bosons ϕ defined by the lagrangian:

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \frac{1}{2}[\partial_\mu\bar{\psi}\gamma^\mu\psi - \bar{\psi}\gamma^\mu\partial_\mu\psi] - m\bar{\psi}[1 - \phi/\phi_o]\psi \quad (2.4)$$

The self interaction of the scalar field is parametrised in $V(\phi)$ as:

$$V(\phi) = \frac{1}{2}m_\phi\phi^2[1 - \phi/\phi_o]^2 + B[4(\phi/\phi_o)^3 - 3(\phi/\phi_o)^4] \quad (2.5)$$

A large number N of fermions can get trapped inside a spherical volume of radius R_o if the energy of a fermion in the interior is chosen to be less than its on - shell energy outside. Apart from the surface term, we can work in the mean field approximation with $\phi = \phi_o$ inside and $\phi = 0$ outside. The two regions are separated by a shell of thickness $\approx (m_\phi)^{-1}$ and has a surface energy density $s \approx m_\phi\phi_o^2/6$. The interior fermions may either have an energy distribution of a degenerate fermion gas [with a chemical potential μ] [18(a)] or may just be a trapped relativistic gas with the distribution described by a temperature T [18(b)]. The total energy of a NTS has contributions from: (1) the surface tension energy $E_s \approx sR_o^2$, (2) the energy of the fermions $E_f \approx N^{4/3}/R$ and, (3) the volume energy [$E_V \approx V(\phi_{in})R^3$]. $V(\phi_{in})$ may be positive, negative or vanishing. For the degenerate case [$V(\phi_{in}) = 0$], a NTS has total mass constrained by stability against gravitational collapse to a value determined by the surface tension s . The soliton mass, obtained by minimising the total energy, is just $M = 12\pi sR_o^2$. The radius ought to be bounded from below by $2GM$, giving a critical mass $M_c \leq [48\pi G^2 s]^{-1}$. For $s \approx (30\text{Gev})^3$, this mass bound is $M_c \approx 10^{15}M_\odot$ and the radius lower bound is $R_c \approx 10^2$ light years. For $s \approx (\text{Mev})^3$ and the number of fermions $\approx 10^{75}$, the size of the NTS is of the order of tens of kilo parsecs while it is still away from the Schwarzschild bound. As one moves away from $V(\phi_{in}) = 0$, the size of a stable soliton gets drastically effected. For $V(\phi_{in}) > 0$, the critical mass determined by the onset of gravitational collapse is precipitously reduced and for $V(\phi_{in}) < 0$, a stable NTS can exist only for very small fermion number.

We shall now demonstrate the existence of bounce solutions in the theory described by eqn(2.3). Taking cue from the above flat space analysis, one needs a low effective surface tension $s \leq (\text{Mev})^3$ to ensure a large Schwarzschild bound and be able to keep away from it even for large enough NTS's. We shall consistently consider a low enough fermion number NTS so that gravitation is just a small perturbation over the essentially a flat spacetime analysis reported above.

We look for a spherically symmetric, static solution described by the metric:

$$ds^2 = e^{2u(r)}dt^2 - e^{2\bar{v}(r)}dr^2 - r^2[d\theta^2 + \sin^2\theta d\varphi^2] \quad (2.6)$$

The configuration that is sought would have $\phi(r)$ locked to an almost constant value inside a spherical domain and transiting to the exterior region across a thin surface. As stated earlier, the scalar field gives mass to the fermions as prescribed in eqn(2.4). However, we shall consider a general effective potential $V(\phi)$ that is bounded from below [figure 1] and having a zero at $\phi_{in}^o < \phi_o$. We consider a relativistic fermi gas trapped in the sphere with the interior scalar field taking values in an open interval containing ϕ_{in}^o : i.e. $\phi_{in} \in (\phi_{in}^o - |\delta|, \phi_o)$. An NTS that we look for has the scalar field held to a value ϕ_{in} in the interior of a sphere and makes a transition to $\phi = 0$ just outside the sphere. The fermion density is expected to fall as ϕ falls across the surface as the fermion effective mass increases with decreasing ϕ . To demonstrate the existence of NTS's it is sufficient to

ignore the ϕ - fermion coupling and the matter stress energy across the boundary. Thus across the boundary, taking the trace of [A.1] enables us to eliminate the scalar curvature in [A.2]. The expressions for the scalar curvature and the equation for $\phi(r)$ reduce to:

$$R[U - 3U'^2] = \frac{1}{2}[+e^{2\bar{v}}(\phi_{,r})^2 - 6e^{2\bar{v}}\phi_{,r}\phi_{,r}U'' + 4V(\phi) - 6U'V'] \quad (2.7)$$

$$[1 - \frac{3U'^2}{U}]\nabla^2\phi - (\phi_{,r})^2e^{2\bar{v}}\frac{U'}{U}[\frac{1}{2} - 3U''] + V' - \frac{2U'V}{U} = 0 \quad (2.8)$$

Here

$$\nabla^2\phi = -(e^{2\bar{v}}\sqrt{-g}\phi_{,r})_{,r}/\sqrt{-g} \quad (2.9)$$

Consider $W(\phi)$ and $F(\phi)$ defined by

$$W'(1 - \frac{3U'^2}{U}) \equiv V' - \frac{2U'}{U}V \quad (2.10a)$$

$$F(U) \equiv (1 - \frac{3U'^2}{U})^{-1}(\frac{1}{2} - 3U'')\frac{U'}{U} \quad (2.10b)$$

W has a minimum at $\phi = 0$ for arbitrary (but bounded) $V(0)$, on account of the diverging $U(\phi \rightarrow 0)$. For divergent U , U' , we choose U so that $(1 - 3U'^2/U)$ does not change its (negative) sign in the domain $(0, \phi_o)$. It would merely require U'/U to be sufficiently large in this domain. For the same reason, it easy to ensure F to be negative in the same domain, for W to have a profile as outlined in Fig. 2, and finally, for the satisfaction of the following *sufficient* condition for the existence of NTS's:

$$\int_{\epsilon}^{\phi} F[U(\phi)]d\phi \rightarrow \infty \text{ if } \epsilon \rightarrow 0 \quad (2.11)$$

As long as one stays away from the Schwarzschild bound, spacetime can be considered to be flat to a good approximation. In this flat - space limit, the wave eqn(2.9) for the scalar field reduces to:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + F(U)[\frac{d\phi}{dr}]^2 = W' \quad (2.12)$$

This is an equation of a particle with *position* ϕ at *time* r moving in a potential $-W$. The second and the third terms are time and velocity dependent damping terms. The proof of existence can be constructed along the lines of “overshoot - undershoot” arguments used by Coleman [27]. For a sufficiently large U'/U , W is sufficiently flat. In the interior of a soliton of large radius R_o , [large enough so that the second term is negligible in our analysis], the scalar field is held to a constant value dependent on the fermion density parameter S [eqn(B.13)]. At R_o , the field falls rapidly with a simultaneous precipitous fall of S . For large gradient of the scalar field, just outside R_o , the dominant term in eqn(2.12) is the third term. The divergence of U at $\phi = 0$ and eqn(2.11) ensure that the scalar field would go all the way to $\phi = 0$ for an initial arbitrarily large scalar field gradient $[\phi'_{in}]$ as

$$\frac{d\phi}{dr} = (\frac{d\phi}{dr})_o \exp[\int_{\phi}^{\phi_{in}} F[U(\phi)]d\phi] \quad (2.13)$$

This demonstrates sufficient and not necessary conditions for NTS's though we have not found an argument that could demonstrate their existence in general.

For a given conserved fermion number and surface term of an NTS, its existence is assured by eqn(2.11) provided the initial gradient of the scalar field at R_o is large enough. Its size is constrained by stability. As shown in the flat space case, the larger a NTS is, the closer should the interior potential be to zero. Thus all large NTS's have ϕ approaching ϕ_{in}^o in their interiors. The effective gravitational constant inside all large NTS's dynamically approach $[U(\phi_{in}^o)]^{-1}$. The effective cosmological constant goes to zero inside large NTS's and is identically zero outside as $U(\phi \rightarrow 0)$ diverges.

Similar constructions are possible in more complicated NMC's. Consider for example a particular two component ST theory invariant under $SO(2)$ rotations in the internal ϕ_1, ϕ_2 space. The action being described by eqn[2.3] with $U(|\phi|)$ and $V(|\phi|)$ being functions of ϕ_1, ϕ_2 through the $SO(2)$ invariant $|\phi| \equiv \sqrt{\phi_1^2 + \phi_2^2}$.

The invariance of the theory under $SO(2)$ rotations in $[\phi_1, \phi_2]$ space implies the conservation of any non - topological charge for a configuration having a compact support on any spacelike hypersurface Σ . The conserved current and the consequent conserved charge are given by:

$$J_\mu = \phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1 \quad (2.14)$$

$$Q = \int_\Sigma d\Sigma J^o \quad (2.15)$$

In flat spacetime, charge conservation plays a key role in the existence and stability of non - trivial, NTS's. These are "Q - balls" suggested by Coleman [27]. For these solutions we have $\phi = \phi(r) = \phi_{in}$ for radial coordinate r less than some radius R_o , and ϕ quickly going to zero outside this radius. The two regions are separated by a transition zone of thickness independent of the total charge Q . The total charge and the internal energy of the soliton are degenerate for a given volume. The surface energy is proportional to the surface area of the solution. Thus the total energy for a given charge of the solution is minimum for a sphere. There is no limit to the size of these solutions. The size is determined, however, by the total charge of the solution. Taking the cue from the flat space solutions we look for time dependent solutions in which the scalar field rotates in the internal ϕ_1, ϕ_2 space with angular frequency ω

$$\phi_1 = \phi(r) \sin(\omega t); \phi_2 = \phi(r) \cos(\omega t) \quad (2.16)$$

For a spherically symmetric, static solution described by the metric:

$$ds^2 = g_{oo}(r) dt^2 - g_{rr}(r) dr^2 - r^2 [d\theta^2 + \sin^2 \theta d\phi^2] \quad (2.17)$$

the configuration that is sought would have $\phi(r)$ locked to an almost constant value inside a spherical domain and transiting to the exterior region across a thin surface. It is straightforward to generalise [A.1] for this $SO(2)$ invariant theory. The expressions for the scalar curvature and the equation for $\phi(r)$ reduce to:

$$R[U - 3U'^2] =$$

$$\frac{1}{2}[T_{m\alpha}^\alpha - \omega^2 g^{oo}\phi^2 + g^{rr}(\phi_{,r})^2 - 6g^{rr}\phi_{,r}\phi_{,r}U'' + 6g^{oo}\omega^2\phi U' + 4V(\phi) - 6U'V'] \quad (2.18)$$

$$\begin{aligned} & [1 - \frac{3U'^2}{U}]\nabla^2\phi - \omega^2\phi g^{oo}[1 - \frac{U'\phi}{2U}] + V' \\ & - \frac{U'}{2U}T_{m\alpha}^\alpha - (\phi_{,r})^2 g^{rr} \frac{U'}{U}[\frac{1}{2} - 3U''] - \frac{2U'V}{U} = 0 \end{aligned} \quad (2.19)$$

Here

$$\nabla^2\phi = -(g^{rr}\sqrt{-g}\phi_{,r})_{,r}/\sqrt{-g} \quad (2.20)$$

With W and F defined as before, the flat - space limit for the wave eqn(2.19) of the scalar field reduces to:

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + F(U)\frac{d\phi^2}{dr} = \hat{W}' \quad (2.21)$$

where

$$\hat{W}' \equiv W' + (1 - \frac{3U'^2}{U})^{-1}[-\omega^2\phi(1 - \frac{U'}{2U})] \quad (2.22)$$

The existence of NTS's can be demonstrated as before. Interestingly, the interior metric has an exact solution: Plugging the metric (2.17) into the field equation (A.1) for a uniform scalar field in the interior gives the following regular solution:

$$ds^2 = \frac{\omega^2\phi_{in}^2}{V(\phi_{in})}dt^2 - \frac{dr^2}{1 - Cr^2} - r^2[d\theta^2 + \sin^2\theta d\phi^2] \quad (2.23)$$

with $C \equiv V(\phi_{in})/4U(\phi_{in})$. Expressions for the conserved charge and the internal energy of such a solution are:

$$Q = \omega\phi_{in}^2 g^{oo}v = vV/\omega \quad (2.24)$$

$$E = 2vV \quad (2.25)$$

Here v is the invariant three volume of sphere of radius r . To get the total energy one would also add the surface term which would be proportional to the area of the sphere. We have considered the NTS to be spherically symmetric. Aside from the surface term, the total energy of a NTS for a given charge is degenerate in volume. The assumption of spherical symmetry therefore requires the contribution to the surface energy to be positive definite. The total energy would then be minimised for a sphere and be consistent with the assumption of spherical symmetry.

Once a NTS is formed with the scalar field arrested at a value ϕ_{in} in the interior, the configuration would evolve to a state of lower energy by changing ϕ_{in} . The energy could be transmitted to a changing energy of rotation of the scalar field in internal space and also to the expansion of the surface wall. A drift of the scalar effective potential $V(\phi)$ to a vanishingly small value while conserving the total charge must be accompanied by the volume v blowing up to infinity. This is the only way one could preserve the metric signature as $V(\phi) \rightarrow 0$ while conserving the charge. [For a bounded g_{oo} as $V \rightarrow 0$,

$\omega \longrightarrow \sqrt{V}$. For conserved Q , this means $v \longrightarrow \infty$ as $V \longrightarrow 0$]. The larger the NTS becomes, the longer would be the time expected to synchronise the rotation of the field throughout the interior and the slower would be any further drift to $V(\phi) \longrightarrow 0$. Thus a NTS would approach $V(\phi) \longrightarrow 0$ in infinite time.

Thus in the quasi - static approximation described above, the solution would approach a flat configuration in the interior. The effective gravitational constant would approach a value $[U(\phi_o)]^{-1}$ irrespective of the initial value that the NTS may be borne with. As before, this would dynamically generate the universality of the induced gravitational constant and the vanishing of the effective cosmological constant.

Section III

A “toy” cosmology based on the Milne model [eqn(2.1)] has characteristic features: (i) With the absence of the flatness [fine tuning] and the horizon large scale homogeneity and isotropy can be dynamically generated. (ii) The standard classical cosmological tests, viz.: the number count, angular diameter and the luminosity distance variation with redshift are comfortably consistent in such a cosmology. Kolb [16] has demonstrated concordance of classical cosmological tests with a coasting cosmology and we [16] have extended his analysis to the Milne model outlined above. The first two tests are quite sensitive to models of galactic evolution and for this reason have (of late) fallen into disfavour as reliable indicators of a viable model. However the magnitude - redshift measurements on SN 1A have a great degree of concordance with $\Omega_\Lambda = \Omega_M = 0$ [10]. (iii) With the scale factor evolving linearly with time, the Hubble parameter is precisely the inverse of the age t . Thus the age of the universe inferred from a measurement of the Hubble parameter is 1.5 times the age inferred by the same measurement in standard matter dominated model. Such a cosmology promises consistency with an older universe. (iv) The deceleration parameter is predicted to vanish.

A linear evolution of the scale factor would radically effect nucleosynthesis in the early universe. Surprisingly, one can still expect the following scenario to go through [28]. Energy conservation, in a period where the baryon entropy ratio does not change, enables the distribution of photons to be described by an effective temperature T that scales as $a(t)T = \text{constant}$ [29]. With the age of the universe $\approx 10^{10}$ years, and $T \approx 2.7K$, one concludes that the age of the universe at $T \approx 10^{10}K$ would be of the order of years [rather than seconds as in standard cosmology]. The universe would take some 10^3 years to cool to 10^7K . With such a low rate of expansion, weak interactions remain in equilibrium for temperatures as low as 10^8K . The onset of nucleosynthesis is determined by the temperature at which deuterium burning into heavier elements becomes a more efficient mode for destruction of neutrons than neutron decay. This temperature is completely determined by the baryon entropy ratio and is around 10^9K for interesting values. With weak interactions still in equilibrium at this temperature, the neutron - proton ratio keeps falling as $n/p \approx \exp[-15/T_9]$. There would hardly be any neutrons left when nucleosynthesis commences at $T_9 \approx 1$. However, as weak interactions are still in equilibrium, once nucleosynthesis commences, inverse beta decay would convert protons into neutrons and pump them into the nucleosynthesis channel. It turns out [28] that for baryon entropy ra-

tio 10^{-8} , there would just be enough neutrons produced, after nucleosynthesis commences, to give $\approx 23.9\%$ ${}^4\text{He}$ and metallicity some 10^8 times the metallicity produced in the early universe in the standard scenario. This metallicity is of the same order of magnitude as seen in lowest metallicity objects. The bad news is that the residual deuterium that we get is rather low. We are exploring the possibility of having a non uniform baryon entropy ratio and spallation of ${}^4\text{He}$ deficient baryons onto a ${}^4\text{He}$ rich clouds as a mechanism to get the right deuterium while maintaining a low value for Lithium and other light elements.

A Milne model within the framework of a divergent NMC would have a vanishing cosmological constant. Large enough gravitating NTS domains would require a conserved fermion number in the Lee - Wick construction reported here. A value $N_f \approx 10^{75}$ for $s \approx (\text{Mev})^3$ can give a NTS of a size of tens of kilo parsecs. Such an N_f is of the same order as the relic background neutrons / photons in the universe. Thus a fermion species that decouples very early in the universe and which has a Higgs Coupling as described in eqn(2.4), would be sufficient to provide N_f for gravitating domains as large as a Halo of a typical large structure (galaxy / local group etc.).

How would such large NTS domains arise in the first place ? The lack of a definitive answer to this question is the reason for referring to the idea as a “toy” model. Small NTS’s could form in the early universe as a condensate of the scalar field evolves in the universe. A large number of expanding $\phi = 0$ pockets can constrain the condensate to colliding walls where the NTS’s would be pinched off as percolation of the entire condensate would be halted by the energetics of the fermi - higgs coupling. Larger NTS’s would form by collisions of smaller configurations. Such a possibility would have a characteristic signature on the microwave background [CMB] anisotropy. A distribution of small NTS’s at $z \approx 1100$ (the surface of last scattering), that would later form a large NTS, would appear hotter than the background. The number of such distributions that would be picked up in a typical detector’s beam-width should correspond to the number of large structures at the present epoch in a scale corresponding to the beam width size. The rms fluctuation in temperature, $\langle \Delta T/T \rangle$, would be determined by the inverse root of the average number of structures picked up by the beam width. The variation of the rms $\langle \Delta T/T \rangle$ with the beam width size would be the characteristic feature of such a model [30].

Constraints on a NTS from lensing can be restricted to the form of the metric [eqns(B.2, B.16)] just inside the boundary (R_o)[31]. A metric of the form:

$$ds^2 = dt^2 - dr^2 - r^2[d\theta^2 + \sin^2\theta d\varphi^2] \quad r > R_o \quad (3.1)$$

and

$$ds^2 \approx e^{2u_o} dt^2 - dr^2 - r^2[d\theta^2 + \sin^2\theta d\varphi^2] \quad r < R_o \quad (3.2)$$

can be used to constrain u_o from lensing data. It can be seen that a value $u_o \approx 10^{-3}$ is consistent for most lensing objects that we have considered. Such a value could be put to good use, as far nucleosynthesis is concerned, as it enables a baryon at rest outside the ball to acquire a kinetic energy $\approx \text{Mev}$ as it goes inside the ball. In an inhomogeneous model - having a different baryon entropy ratio in the interior and exterior of NTS’s, one could easily have a ${}^4\text{He}$, metal rich, interior, and a proton rich exterior, by the time the universe cools to temperatures below $\approx 10^7 K$. At lower temperatures, the proton rich clouds in the exterior can spall over the interior matter and lead to deuterium formation [28].

IV. Conclusion

There are two distinct aspects of the work presented here. First is the viability, advantages and consequences of a coasting cosmology. The second aspect is the possible realisation of such a coasting. The absence of the horizon, flatness and age problems distinguish a coasting cosmology.

We have demonstrated that in a whole class of scalar tensor theories in which the non - minimal coupling diverges and for which the classical effective potential vanishes at some point, classical scalar field condensates can occur as NTS's. The effective gravitational constant inside all large domains would approach a universal value and the effective cosmological constant would drift to zero. The dynamical tuning of the effective cosmological constant to a small value and the effective gravitational constant to a universal value are compelling features - enough to explore the possibility of raising the toy model described here to the status of a viable cosmology.

The cosmology described in this article essentially requires the scale factor to coast linearly with time. The particular model outlined here is deficient in several respects. Firstly we have treated the scalar field as purely classical. The stability of the NTS against decay has not been studied. We feel that the peculiar behaviour of the NMC at $\phi = 0$ would prevent any stable ϕ particle states to materialise near $\phi = 0$.

The idea of exploring NMC for the purpose of getting stress energy of the scalar field condensate to compensate the cosmological constant was also suggested by Dolgov[4]. It was shown however that the NMC itself diverged. It was indeed suggested that a spatial variation of the scalar field - and hence a gravitational constant be explored for a non - trivial model. What we have shown is that in a model where the NMC diverges over most of the space and is finite over compact domains, the compact domains can be expected to inflate. Conservation of charge and energy would then ensure that the effective cosmological constant approaches zero inside these domains and identically vanishes outside. Instead of working with a NMC that diverges as a function of ϕ as $\phi \rightarrow 0$, one could equally well work with $U(\phi) = \epsilon\phi^2$ in a model where $V(\phi \rightarrow \infty) = \text{constant}$. In such a model, as discovered by Dolgov [23], there are solutions that has the scalar field rising to infinity as a function of time. If $V(\phi)$ has a zero then, with a judicious Higgs coupling of a fermion as described in this article, one would get results similar to those obtained.

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Appendix A

We describe properties of the scalar tensor theory and derive expressions for a conserved pseudo energy tensor for the theory. The theory is described by the action:

$$S = \int \sqrt{-g} d^4x [U(\phi)R + L_\phi + L_m] \equiv \int \sqrt{-g} d^4x [U(\phi)R + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) + L_m] \quad (2.3)$$

Here ϕ is the scalar field, $U(\phi)R$ is a non - minimal coupling of the scalar field with the scalar curvature, and $V(\phi)$ is the scalar effective potential. L_m consists of a fermion field ψ , together with its Higgs coupling with ϕ , and L_w (the rest of the matter lagrangian) independent of ϕ and ψ :

$$L_m \equiv L_\psi + L_{\psi,\phi} + L_w \equiv \frac{1}{2}[\bar{\psi}\overleftarrow{D}_\mu\gamma^\mu\psi - \bar{\psi}\gamma^\mu\overrightarrow{D}_\mu\psi] - m(1 - \frac{\phi}{\phi_o})\bar{\psi}\psi + L_w$$

Here D_μ is the spin covariant derivative [see eg. [33]]

$$\overrightarrow{D}_\mu\psi = (\partial_\mu + \Gamma_\mu)\psi$$

$$\bar{\psi}\overleftarrow{D}_\mu = (\partial_\mu\bar{\psi} - \bar{\psi}\Gamma_\mu)$$

Γ_μ are the spin connection [Fock - Ivanenko] coefficients defined by:

$$D_\nu\gamma_\mu \equiv \partial_\nu\gamma_\mu - \Gamma_{\mu\nu}^\alpha\gamma_\alpha + [\Gamma_\nu, \gamma_\mu]$$

Requiring the action to be stationary under variations of the metric tensor and the fields ϕ, ψ , gives the equations of motion:

$$U(\phi)[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R] = -\frac{1}{2}[T_w^{\mu\nu} + T_\phi^{\mu\nu} + T_{\phi,\psi}^{\mu\nu} + T_\psi^{\mu\nu} + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu}U(\phi)]_{;\lambda}^\lambda \quad (A.1)$$

$$g^{\mu\nu}\phi_{;\mu;\nu} + \frac{\partial V}{\partial\phi} - R\frac{\partial U}{\partial\phi} - \frac{m}{\phi_o}\bar{\psi}\psi = 0 \quad (A.2(a))$$

$$\gamma^\mu D_\mu\psi + m(1 - \frac{\phi}{\phi_o})\psi = 0 \quad (A.2(b))$$

$$D_\mu\bar{\psi}\gamma^\mu - m(1 - \frac{\phi}{\phi_o})\bar{\psi} = 0 \quad (A.2(c))$$

Here $T_w^{\mu\nu}$, $T_\psi^{\mu\nu}$ and $T_{\phi,\psi}^{\mu\nu}$ are the energy momentum tensors constructed from L_w and $L_\psi + L_{\psi,\phi}$ respectively, and

$$T_\phi^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - g^{\mu\nu}[\frac{1}{2}\partial^\lambda\phi\partial_\lambda\phi - V(\phi)] \quad (A.3)$$

L_w is independent of ϕ . To examine this theory viz - a - viz the equivalence principle, we have to explore conditions under which $T_{;\nu}^{\mu\nu} = 0$. Eqns.(2(a), 2(b)) show that the portion of lagrangian $L_\psi + L_{\psi,\phi}$ is null. The stress tensor is given by the following generalisation of the familiar flat spacetime expression [see eg. [34]]:

$$\Theta_\nu^\mu \equiv T_{\psi\nu}^\mu + T_{\psi,\phi,\nu}^\mu = -\frac{1}{2}[\bar{\psi}\overleftarrow{D}_\nu\gamma^\mu\psi - \bar{\psi}\gamma^\mu\overrightarrow{D}_\nu\psi] \quad (A.4)$$

When applied to any spinor or any “spin - matrix” such as the Dirac matrices, one replaces the ordinary derivative by the spin - covariant derivative [33]. The covariant divergence of [A.4] is easily seen to reduce to:

$$\Theta_{\nu;\mu}^{\mu} = \frac{m}{\phi_o} \partial_{\nu} \phi \bar{\psi} \psi \quad (A.5)$$

Thus there is a violation of equivalence principal as far as the fermi field is concerned. However, in a region where the scalar field gradient, $\partial_{\mu} \phi$, vanishes, the covariant divergence of the fermion field stress tensor vanishes. For the rest of the matter fields, the equivalence principal holds strictly, i.e.: $T_{w;\nu}^{\mu\nu} = 0$. To see this, consider the covariant divergence of [A.1]. From the contracted Bianchi identity satisfied by the Einstein tensor, we get

$$U(\phi)_{,\nu} [R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R] = -\frac{1}{2} [T_{w;\nu}^{\mu\nu} + t_{;\nu}^{\mu\nu} + \Theta_{;\nu}^{\mu\nu}] \quad (A.6)$$

with

$$t^{\mu\nu} \equiv T_{\phi}^{\mu\nu} + 2U(\phi)^{;\mu;\nu} - 2g^{\mu\nu} U(\phi)^{;\lambda}_{;\lambda} \quad (A.7)$$

Using the identity: $U(\phi)^{;\rho} R_{\rho\alpha} = U(\phi)^{;\lambda}_{;\lambda;\alpha} - U(\phi)^{;\lambda}_{;\alpha;\lambda}$ and the eqn(A.5), this reduces to

$$-\frac{1}{2} U(\phi)^{;\mu} R] = -\frac{1}{2} [T_{w;\nu}^{\mu\nu} + T_{\phi;\nu}^{\mu\nu} + \partial^{\mu} (\frac{m}{\phi_o}) \bar{\psi} \psi]$$

Finally, using the equation of motion for the scalar field [A.2a], all the ϕ dependent terms cancel the left hand side - giving the vanishing of the covariant divergence of the (w-) matter stress energy tensor.

One can find the expression for a conserved pseudo energy momentum tensor that would be conserved. To achieve this we proceed to express the vanishing covariant divergence of the matter stress energy tensor as:

$$[\sqrt{-g} T_{m\mu}^{\nu}]_{,\nu} - \frac{1}{2} g_{\tau\beta,\mu} \sqrt{-g} T_m^{\tau\beta} = 0 \quad (A.8)$$

To cast the LHS of the above equation into a total ordinary divergence one has to seek a representation of the second quantity in terms of an ordinary total divergence. This can be done as follows. First we make use of the equation of motion (A.1) to express the matter stress energy tensor in terms of the other fields and the metric -dependent quantities: $T_w^{\tau\beta} \equiv -t^{\tau\beta} - \Theta^{\tau\beta} - 2U(\phi) G^{\tau\beta}$. Second, note that the right hand side of this expression is merely the variational derivative of

$$J \equiv 2 \int \sqrt{-g} d^4x [U(\phi) R + L_{\phi} + L_{\psi} + L_{\psi,\phi}] \quad (A.9)$$

under variations of the metric tensor, with boundary conditions that require the vanishing of metric and its first derivative variations on the boundary of a (3+1) - dimensional manifold over which this integral has been taken. Consider the standard decomposition of

$\sqrt{-g}R$ into a pure divergence term and a simple expression involving only the metric and its first derivatives:

$$\sqrt{-g}R = A + [\sqrt{-g}g^{\sigma\rho}\Gamma_{\sigma\alpha}^\alpha]_{,\rho} - [\sqrt{-g}g^{\sigma\rho}\Gamma_{\sigma\rho}^\alpha]_{,\alpha} \quad (A.10)$$

with

$$A \equiv \sqrt{-g}g^{\sigma\rho}[\Gamma_{\sigma\rho}^\alpha]\Gamma_{\alpha\beta}^\beta - \Gamma_{\beta\rho}^\alpha[\Gamma_{\alpha\sigma}^\beta] \quad (A.11)$$

It follows that the functional derivative of J with respect to the metric tensor is the same as that of

$$H \equiv \int d^4x [B + \sqrt{-g}(L_\phi + L_\psi + L_{\psi,\phi})] \quad (A.12)$$

where

$$B \equiv [UA - \sqrt{-g}g^{\sigma\rho}\Gamma_{\sigma\alpha}^\alpha U_{,\rho} + \sqrt{-g}g^{\sigma\rho}\Gamma_{\sigma\rho}^\alpha U_{,\alpha}] \quad (A.13)$$

In other words

$$\begin{aligned} & \sqrt{-g}UG_{\mu\nu} + \sqrt{-g}[U_{;\mu;\nu} - g_{\mu\nu}U_{;\alpha}^{\alpha}] + \frac{1}{2}\sqrt{-g}T_{(\phi+\psi)\mu\nu} \\ &= \frac{\partial}{\partial g^{\mu\nu}}[B + \sqrt{-g}L_{\phi+\psi}] - [\frac{\partial(B + \sqrt{-g}L_{\phi+\psi})}{\partial g^{\mu\nu}_{,\lambda}}]_{,\lambda} \end{aligned} \quad (A.14)$$

This is just a generalisation of the standard procedure in GTR [32]. Definining $\hat{B} \equiv B + \sqrt{-g}L_{\phi+\psi}$, the expression for the ordinary derivative of \hat{B} and the field equation for the fields ϕ, ψ easily enable us to express the second term in eqn(A.8) as a total divergence. This gives

$$[\sqrt{-g}T_{m\mu}^\nu - \hat{B}\delta_\mu^\nu - \frac{\partial\hat{B}}{\partial g_{,\nu}^{\tau\beta}}g_{,\mu}^{\tau\beta} - \frac{\partial\hat{B}}{\partial\phi_{,\nu}}\phi_{,\mu} - \frac{\partial\hat{B}}{\partial\psi_{,\nu}}\psi_{,\mu} - \bar{\psi}_{,\mu}\frac{\partial\hat{B}}{\partial\bar{\psi}_{,\nu}}]_{,\nu} = 0 \quad (A.15)$$

For $\nu = o$ the expression within the brackets integrated over a spacelike hypersurface is thus invariant under time translations for a distribution of matter and the rest of the terms in (A.15) having a compact support over the surface. This is the expression for the pseudo energy momentum tensor that we seek. The quantity

$$P_\mu \equiv \int_\Sigma d\Sigma [\sqrt{-g}T_{w\mu}^o - \hat{B}\delta_\mu^o - \frac{\partial\hat{B}}{\partial g_{,o}^{\tau\beta}}g_{,\mu}^{\tau\beta} - \frac{\partial\hat{B}}{\partial\phi_{,o}}\phi_{,\mu} - \bar{\psi}_{,\mu}\frac{\partial\hat{B}}{\partial\bar{\psi}_{,o}} - \frac{\partial\hat{B}}{\partial\psi_{,o}}\psi_{,\mu}] \quad (A.16)$$

evaluated on a constant spacelike hypersurface Σ , is thus conserved. This may be viewed as the generalisation of the energy momentum four vector for the scalar - tensor theory described by eqn[2.3]. The formalism presented here is general and can be used to determine the energy momentum four vector for any Brans - Dicke theory in particular. As in standard general relativity, P_μ is not a generally covariant four vector as A and B are not scalar densities. The intrinsic non - covariance of the energy momentum density of the gravitational field has its origin in the intimate connection between geometry and the

gravitational field. Had the expression been covariant, one could always have gone into a preferred [freely - falling] frame to ensure vanishing of an arbitrary localised gravitational field.

This expression for the energy is sufficient for the present article. For a $U(\phi)$ that is well behaved [bounded], it is possible to reduce the energy as an integral over a two sphere. This is not case for the present article [$U(\phi)$ has been chosen to diverge]. [see Bose, Lohiya– for the reduction].

Appendix B

We derive the form for the metric of a NTS satisfying a “weak field approximation” that would justify retaining only a first order deviation from a flat metric. The metric can be expressed in terms of the spherical [Schwarzschild] coordinates:

$$ds^2 = e^{2u} dt^2 - e^{2\bar{v}} dr^2 - r^2 [d\theta^2 + \sin^2 \theta d\varphi^2] \quad (B.1)$$

or in terms of isotropic coordinates:

$$ds^2 = e^{2u} dt^2 - e^{2v} (d\rho^2 + \rho^2 d\theta^2 + \rho^2 \sin^2 \theta d\varphi^2) \quad (B.2)$$

related to each other by:

$$r = \rho e^v \quad B.3$$

We look for a solution describing the scalar field trapped to a value $\phi = \phi_{in}$ in the interior of a sphere of radius R_o and making a transition across a thin surface to $\phi = 0$ outside. The fermi gas trapped inside the soliton is described by the familiar distribution in momentum space: n_k , k being the momentum measured in an appropriate local frame that depends on r or ρ . The fermion energy density is given by [18]:

$$W = \frac{2}{8\pi^3} \int d^3 k n_k \epsilon_k \quad (B.4)$$

with $\epsilon_k = \sqrt{k^2 + (m - f\phi_{in})^2}$. The number density ν and the non - vanishing components of fermion stress energy tensor are:

$$\nu = \frac{2}{8\pi^3} \int d^3 k n_k \quad (B.5)$$

$$T_t^t = W$$

$$T_r^r = T_\theta^\theta = T_\varphi^\varphi = T_\rho^\rho \equiv -T = -\frac{2}{8\pi^3} \int d^3 k n_k \frac{k^2}{3\epsilon_k} \quad (B.6)$$

The trace of the stress tensor is just:

$$T_\mu^\mu = W - 3T = (m - f\phi_{in})S \quad (B.7)$$

with S the scalar density:

$$S = \frac{2}{8\pi^3} \int d^3k \frac{n_k}{\epsilon_k} (m - f\phi_{in}) \quad (B.8)$$

Defining $G_{in} \equiv U(\phi_{in})^{-1}$ as the effective interior “gravitational constant”, the metric field equation in the interior can be expressed in the spherical coordinates as:

$$r^2 G_t^t = e^{-2\bar{v}} - 1 - e^{-2\bar{v}} r \frac{d\bar{v}}{dr} = -8\pi G r^2 [W + V(in)] \quad B.9$$

$$r^2 G_r^r = e^{-2\bar{v}} - 1 + e^{-2\bar{v}} r \frac{du}{dr} = 8\pi G r^2 [T - V(in)] \quad B.10$$

$$r^2 G_\theta^\theta = e^{-2\bar{v}} [r^2 \frac{d^2 u}{dr^2} + [1 + r \frac{du}{dr}] r \frac{d}{dr} (u - \bar{v})] = 8\pi G r^2 [T - V(in)] \quad B.11$$

The scalar field satisfies:

$$\phi_{;\mu}^{\mu} + V' - fS - U'R = 0 \quad B.12$$

Taking the trace of the Einstein tensor G_μ^μ in eqns(B.9 - 11), the Ricci scalar R can be substituted in eqn(B.12). The condition for the existence of a $\phi = \phi_{in} = \text{constant}$ for $0 \leq r \leq R_o$ reduces to:

$$V' - fS - 2V \frac{U'}{U} = \frac{U'}{U} (m - f\phi_{in}) S \quad (B.13)$$

For a large enough choice for the gradient of the NMC, this condition can be satisfied in an open interval containing ϕ_{in}^o for any S . The form for the metric in the linear approximation follows from eqns(B.9 - 11). Defining $\hat{C} \equiv 8\pi G[W + V(\phi_{in})]$, we get $\bar{v} = -\hat{C}r^2/6$. The expression for u follows from:

$$2e^{-\bar{v}} r [\frac{du}{dr} + \frac{d\bar{v}}{dr}] = 8\pi G r^2 [T + W] \equiv \tilde{C} r^2 \quad (B.14)$$

Whence, in the linear approximation being followed,

$$\frac{du}{dr} + \frac{d\bar{v}}{dr} = \frac{1}{2} \tilde{C} r \quad (B.14)$$

From the expression derived for \bar{v} , we get:

$$u = u_o + \frac{r^2}{2} [\frac{\tilde{C}}{2} + \frac{\hat{C}}{3}] \quad (B.15)$$

In isotropic coordinates, this transforms to

$$u = u_o + \frac{\rho^2}{2} [\frac{\tilde{C}}{2} + \frac{\hat{C}}{3}] \quad (B.16)$$

and $v = \hat{C}\rho^2/12$. u_o is a small negative constant that determines the rate at which clocks tick at the origin $r = \rho = 0$. This is determined by integrating the field eqn(A.1) from outside the NTS, where u_o vanishes, across the surface of the NTS into the interior and all the way upto the origin. This constant would determine the bending of a null ray as it moves across the surface of the NTS.

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